CHPTER SIX

Non-Newtonian Fluids

6.1 Introduction

For Newtonian fluids a plot of shear stress (τ), against shear rate

(- ***Ϋ=dux/dy***) on Cartesian coordinate is a straight line having a slope equal to the dynamic viscosity (μ). For many fluids a plot of shear stress against shear rate does not give a straight line. These are so-called " Non-Newtonian Fluids". Plots of shear stress against shear rate are experimentally determined using viscometer.

The term viscosity has no meaning for a non-Newtonian fluid unless it is related to a particular shear rate *Ϋ*. ***An apparent viscosity (μa) can be defined as follows: -***

$$\mu_a = \frac{\tau}{\dot{\gamma}}$$

6.2 Types of Non-Newtonian Fluids

There are two types of non-Newtonian fluids: -

1- Time-independent.

2- Time-dependent.

6.2.1 Time-Independent Non-Newtonian Fluids

In this type ***the apparent viscosity depends only on the rate of shear at any particular moment and not on the time for which the shear rate is applied.***

For non-Newtonian fluids the relationship between shear stress and shear rate is more complex and this type can be written as: -

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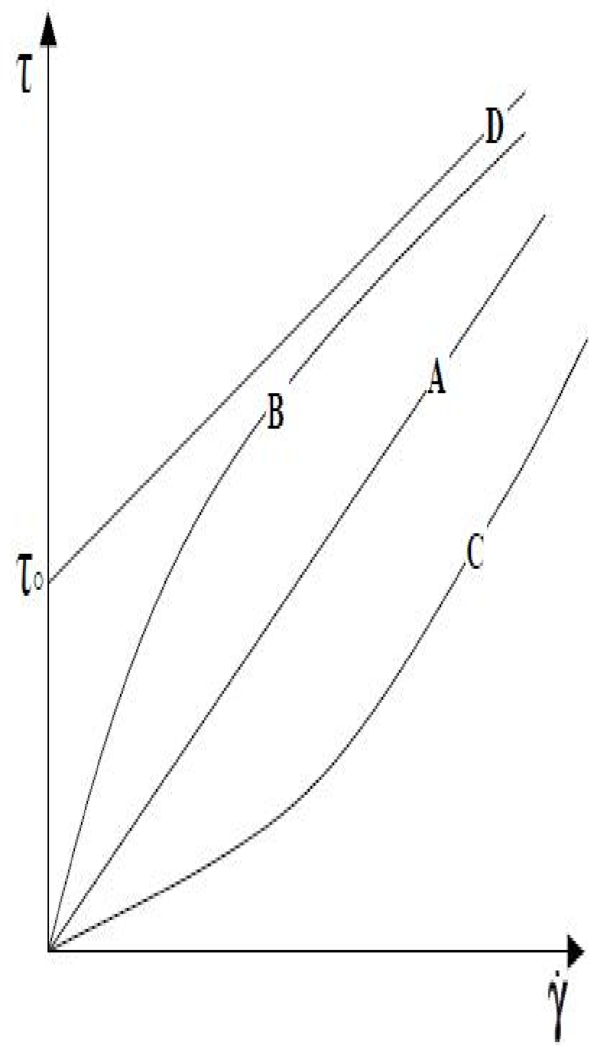
$\tau = k(-\dot{\gamma})^n$ ------------------For power-law fluids

or as

$\tau = \tau_0 + k(-\dot{\gamma})$ ------------------For Bingham plastics fluids

The shape of the flow curve for time-independent fluids in compare with Newtonian fluid is shown in thee Figure, where



A: Newtonian fluids

B: Pseudoplastic fluids [power-law n<1]

Ex. Polymer solution, detergent.

C: Dilatant fluids [power-law n>1]

Ex. Wet beach sand, starch in water.

D: Bingham plastic fluids, it required ($\tau_\circ$)

for initial flow

Ex. Chocolate mixture, soap, sewage sludge, toothpaste.

6.2.2 Time-Dependent Non-Newtonian Fluids

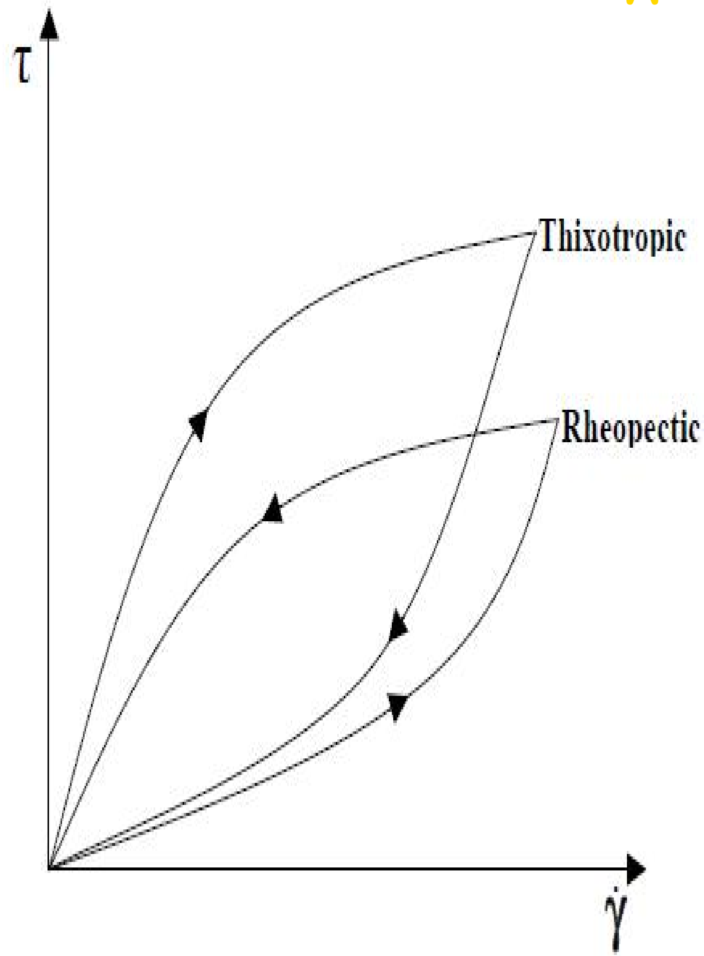
For this type the curves of share stress versus shear rate depend on how long the shear has been active. This type is classified into: -

1- ***Thixotropic Fluids***

Which exhibit a reversible decrease in shear stress and apparent viscosity with time at a constant shear rate. Ex. Paints.

2- ***Rheopectic Fluids***

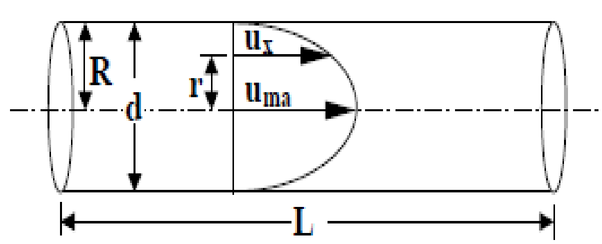
Which exhibit a reversible increase in shear stress and apparent viscosity with time at a constant shear rate. Ex. Gypsum suspensions, bentonite clay.



6.3 Flow Characteristic [8u/d]

The velocity distribution for Newtonian fluid of laminar flow through a circular pipe, as given in chapter four, is given by the following equation;

$$u_x = u_{max}\left[1-\left(\frac{r}{R}\right)^2\right] = 2u\left[1-\left(\frac{r}{R}\right)^2\right]$$



where, u: is the mean (average) linear velocity; u = Q/A

$$\dot{\gamma} = \frac{du_x}{dr} = 2u\left[-\frac{2r}{R}\right] = -4u\frac{r}{R^2}$$

At pipe walls (r = R)

$$\dot{\gamma} = \dot{\gamma}_w = \frac{du_x}{dr}\bigg|_{r=R}$$

$$\dot{\gamma}_w = \frac{-4u}{R} \rightarrow -\dot{\gamma}_w = \frac{8u}{d}$$ Flow characteristic For laminar flow

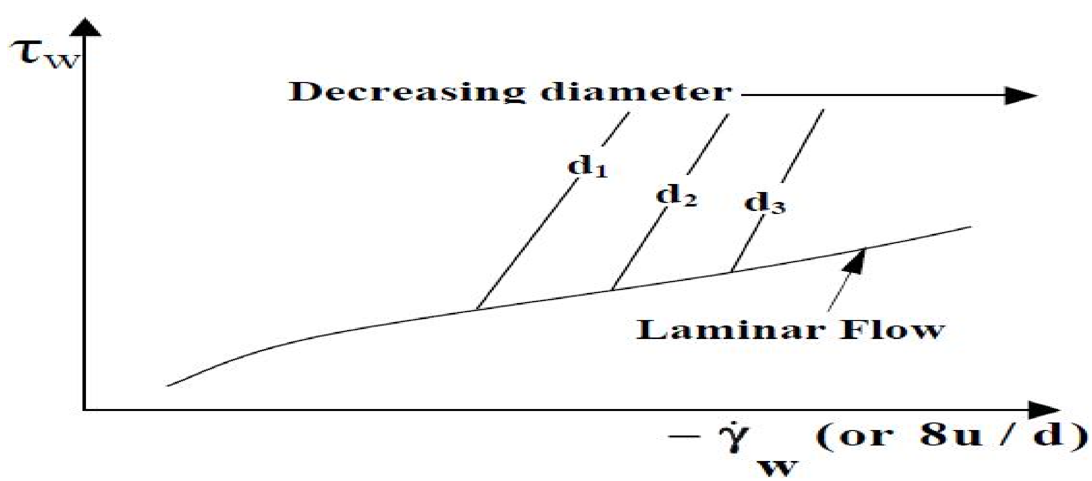
$$\tau_w = -\mu \dot{\gamma}_w = \mu \frac{8u}{d}$$ ---at wall

The force balance on an element of fluid of L length is;

$$\tau_w \pi d L = \frac{\pi}{4} d^2 \Delta P \rightarrow \tau_w = \frac{\Delta P}{4L/d} = \mu \frac{8u}{d} --- (1)$$

this equation for Newtonian fluids

A plot of τ_w or $\Delta P/(4L/d)$ against or $(8u/d)$ is shown in Figure for a typical time independent non-Newtonian fluid flows in a pipe. In laminar flow the plot gives a single line independent of pipe size. In turbulent flow a separate line for each pipe size.



6.4 Flow of Genral Time-Independent Non-Newtonian Fluids

The slope of a log-log plot of shear stress ***at the pipe walls against flow characteristic [8u/d] at any point along the pipe*** ***<u>is the flow behavior index (n')</u>***

$$n' = \frac{d\, ln\tau_w}{d\, ln(-\dot{\gamma}_w)} = \frac{d\, ln\tau_w}{d\, ln(8u/d)} = \frac{d\, ln[\frac{\Delta P}{(4L/d)}]}{d\, ln(8u/d)} --- (2)$$

This equation lead to,

$$\tau_w = \frac{\Delta P}{(4L/d)} = K\acute{p} \left(\frac{8u}{d}\right)^{n'} ------ (3)$$

where, Kp′ and n′ are ***point values for a particular value of the flow characteristic (8u/d).*** or as'

$$\tau_w = \frac{\Delta P}{(4L/d)} = K\acute{p} \left(\frac{8u}{d}\right)^{n'-1} \frac{8u}{d} ------ (4)$$

By the analogy of equation (4) with equation (1), the following equation can be written for non-Newtonian fluids;

$$\tau_w = \frac{\Delta P}{(4L/d)} = (\mu_a)_p \left(\frac{8u}{d}\right) ------ (5)$$

where, $(\mu_a)_P$ is apparent viscosity for pipe flow.

$$\therefore (\mu_a)_p = K\acute{p} \left(\frac{8u}{d}\right)^{n'-1} ----(6)$$

This equation gives a ***point value for the apparent viscosity of non-Newtonian fluid flow through a pipe.***
Reynolds number for the of non-Newtonian fluids can be written as;

$$Re = \frac{\rho u d}{(\mu_a)_p} = \frac{\rho u d}{K\acute{p} \left(\frac{8u}{d}\right)^{n'-1}} ---- (7)$$

$$Re = \frac{\rho u^{2-n'} d^{n'}}{m} ----(8)$$ **where, m = Kp′ ($8^{n'-1}$)**

Equations (7) or (8) gives a ***point value for Re at a particular flow characteristic (8u/d).***

A point value of the basic friction factor (Φ or J_f) or fanning friction factor (*f*) *for* *<u>laminar flow can be obtained from;</u>*

$\Phi = J_f = 8 / Re$ or $f = 16 / Re$ --------------(9)

The pressure drop due to skin friction can be calculated in the same way as for Newtonian fluids,

$-\Delta P_{fs} = 4f\,(L/d)\,(\rho u^2/2)$ ----------------------(10)

Equation (10) is used for laminar and turbulent flow, and the fanning friction factor (*f*) *for turbulent flow of general time independent non-Newtonian fluids in* ***<u>smooth cylindrical pipes can be calculated from;</u>***

$f = a / Re^b$ ----------------------(11)

where, a, and b are function of the flow behavior index (n′)

n′	0.2	0.3	0.4	0.6	0.8	1.0	1.4	2.0
a	0.0646	0.0685	0.0714	0.074	0.0761	0.0779	0.0804	0.0826
b	0.349	0.325	0.307	0.281	0.263	0.25	0.231	0.213

There is another equation to calculate (*f*) *for turbulent flow of time-independent non-Newtonian fluids in smooth cylindrical pipes;*

$$\frac{1}{f^{1/2}} = \frac{4}{(\acute{n})^{0.75}} \log\left[Re f^{\left(1-\frac{\acute{n}}{2}\right)}\right] - \frac{0.4}{(\acute{n})^{1.2}} \text{---} (12)$$

<u>Example -6.1-</u>

A general time-independent non-Newtonian liquid of density 961 kg/m^3 flows steadily with an average velocity of 1.523 m/s through a tube 3.048 m long with an inside diameter of 0.0762 m. For these conditions, the pipe flow consistency coefficient Kp' has a value of 1.48 Pa.s$^{0.3}$ [or 1.48 (kg / m.s^2) s$^{0.3}$]

and n' a value of 0.3. Calculate the values of the apparent viscosity for pipe flow $(\mu_a)_P$, the Reynolds number Re and the pressure drop across the tube, neglecting end effects.

Solution:

Apparent viscosity $$(\mu_a)_p = K\acute{p}\left(\frac{8u}{d}\right)^{\acute{n}-1}$$

= 1.48 (kg/m) $s^{-1.7}$ $[8\,(1.523)/0.0762]^{-0.7} s^{-0.7}$ = 0.04242 kg/m.s (or Pa .s)

$$Re = \frac{\rho u d}{(\mu_a)_p} = \frac{\rho u d}{K\acute{p}\left(\frac{8u}{d}\right)^{\acute{n}-1}}$$ = 961 (1.523)(0.762) / 0.04242

=2629

f = a / Re^b from table n' = 0.3, a = 0.0685, b = 0.325

f = 0.0685 / 2629$^{0.325}$ = 0.005202

$-\Delta P_{fs}$ = 4*f*(*L*/*d*) ($\rho u^2/2$) =

4(0.005202)(3.048/0.0762)[961(1.523)^2/2]

= 927.65 Pa.

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6.5 Flow of Power-Law Fluids in Pipes

Power-law fluids are those in which the shear stress (τ) is related to the shear rate ($\dot{\gamma}$) by this equation;

$$\tau = (\dot{\gamma})^n - - - - (13)$$

For shear stress at a pipe wall (τ_w) and the shear rate at the pipe wall ($\dot{\gamma}_w$), equation (13) becomes;

$$\tau_w = K(\dot{\gamma}_w)^n - - - - - - - (14)$$

Equation (3) gives the relationship between (ΔP) and (8u/d) for general time-independent non-Newtonian fluids.
But for power-law fluids the parameters Kp′ and n′ in equation (3) are no longer point values but remain constant over a range of (8u/d), so that for power-law fluids equation (3) can be written as;

$$\tau_w = \frac{\Delta P}{4L/d} = Kp\left(\frac{8u}{d}\right)^n \quad - - - - - - - (15)$$

where, Kp: is the consistency coefficient for pipe flow.
n: is the power-law inex.

The shear rate at pipe wall for general time-independent non-Newtonian fluids is;

$$\dot{\gamma}_w = \frac{8u}{d}\left(\frac{3\acute{n}+1}{4\acute{n}}\right) - - - - (16)$$

and for *power-law fluids;*

$$\dot{\gamma}_w = \frac{8u}{d}\left(\frac{3n+1}{4n}\right) - - - - (17)$$

Combine equations (14), (15), and (17) to give the relationship between the general consistency coefficient (K) and the consistency coefficient for pipe flow (Kp).

$$Kp = \frac{8u}{d}\left(\frac{3n+1}{4n}\right)^n - - - - (18)$$

The apparent viscosity for power-law fluids in pipe flow

$$\mu_a = Kp\left(\frac{8u}{d}\right)^{n-1} - - - - (19)$$

The Reynolds number for non-Newtonian fluids flow in pipe

$$Re = \frac{\rho u d}{(\mu_a)_{p_a}} \quad ----(20)$$

For power –law fluids flow in pipes the Re can be written either as;

$$Re = \frac{\rho u d}{Kp\left(\frac{8u}{d}\right)^{n-1}} \quad ----(21)$$

or as;

$$Re = \frac{\rho u^{2-n}\, d^n}{m} \quad ----(22)$$

where, m = Kp (8^{n-1}) ----------------------(23)

Example -6.2-

A Power-law liquid of density 961 kg/m^3 flows in steady state with an average velocity of 1.523 m/s through a tube 2.67 m length with an inside diameter of 0.0762 m. For a pipe consistency coefficient of 4.46 Pa.s^n [or 4.46 (kg / m.s^2) s$^{0.3}$], calculate the values of the apparent viscosity for pipe flow (μa)$_P$ in Pa.s, the Reynolds number Re, and the pressure drop across the tube for power-law indices n = 0.3, 0.7, 1.0, and 1.5 respectively.

Solution:

Apparent viscosity $\mu_a = Kp\left(\frac{8u}{d}\right)^{n-1}$

= 4.46 (kg/m) s^{n-2} [8 (1.523)/0.0762]$^{n-1}$s^{n-1}

⇒ (μa)$_P$ = 4.46 (159.9)$^{n-1}$ ---------------(1)

$Re = \frac{\rho u d}{(\mu_a)_p} = \frac{\rho u d}{4.46(159.9)^{n-1}}$ = 961 (1.523)(0.762) / 4.46 (159.9)$^{n-1}$

⇒ Re = 25.006/ (159.9)$^{n-1}$ ---------------------------(2)

$$-\Delta P_{fs} = \left[4f\left(\frac{L}{d}\right)\right]\frac{\rho u^2}{2g}$$
$$= 4\left(\frac{16}{Re}\right)\left(\frac{2.67}{0.0762}\right)[961(1.523)^2/2]$$

$$-\Delta P_{fs} = 99950.56\,(159.9)^{n-1} ------(3)$$

n	$(\mu a)_P$ Eq.(1)	Re Eq.(2)	−Δ Pfs Eq.(3)	(−ΔPfs)New / (−ΔPfs)non-New
0.3	0.1278	872.44	2,865	0.0287
0.7	0.9732	114.6	21,809	0.218
1.0	4.46	25.006	999,50.56	1.0
1.5	56.4	1.9776	1,263,890.541	12.7

6.6 Friction Losses Due to Form Friction in Laminar Flow

Since non-Newtonian power-law fluids flowing in conduits are often in laminar flow because of their usually high effective viscosity, loss in sudden changes of diameter (velocity) and in fittings are important in laminar flow.

1- Kinetic Energy in Laminar Flow

Average kinetic energy per unit mass = $u^2/2\alpha$ [m^2/s^2 or J/kg]

$$\alpha = 1.0 ----- \textit{in turbulent flow}$$
$$\alpha = \frac{(2n+1)(5n+3)}{3(3n+1)^2} --- \textit{in laminar flow}$$

- For Newtonian fluids (n = 1.0) ⇒ α = 1/2 in laminar flow

- For power-law non-Newtonian fluids (n < 1.0 or n > 1.0)

2- Losses in Contraction and Fittings

The frictional pressure losses for non-Newtonian fluids are *very similar to those for Newtonian fluids at the same generalized Reynolds number in laminar and turbulent flow for* ***contractions and also for fittings and valves.***

3- Losses in Sudden Expansion

For a non-Newtonian power-law fluid flow in laminar flow through a sudden expansion from a smaller inside diameter d_1 to a larger inside diameter d_2 of circular cross-sectional area, then the energy losses is

$$F_e = \left[\frac{n+3}{2(5n+3)}\left(\frac{d_1}{d_2}\right)^4 - \left(\frac{d_1}{d_2}\right)^4 + \frac{3(3n+1)}{2(5n+3)}\right]\frac{3n+1}{2n+1}u_1^2$$

6.7 Turbulent Flow and Generalized Friction Factor

The generalized Reynolds number has been defined as

$$Re = \frac{d^{n} u^{2-n} \rho}{m}$$

where, m = Kp′ 8^{n-1} = K 8^{n-1} $(3n+1/4n)^n$

The fanning friction factor is plotted versus the generalized Reynolds number. Since many non-Newtonian power-law fluids have high effective viscosities, they are often in laminar flow. The correction for smooth tube also holds for a rough pipe in laminar flow. For rough pipes with various values of roughness ratio (e/d), the following figure cannot be used for turbulent flow, since it is derived for smooth pipes.

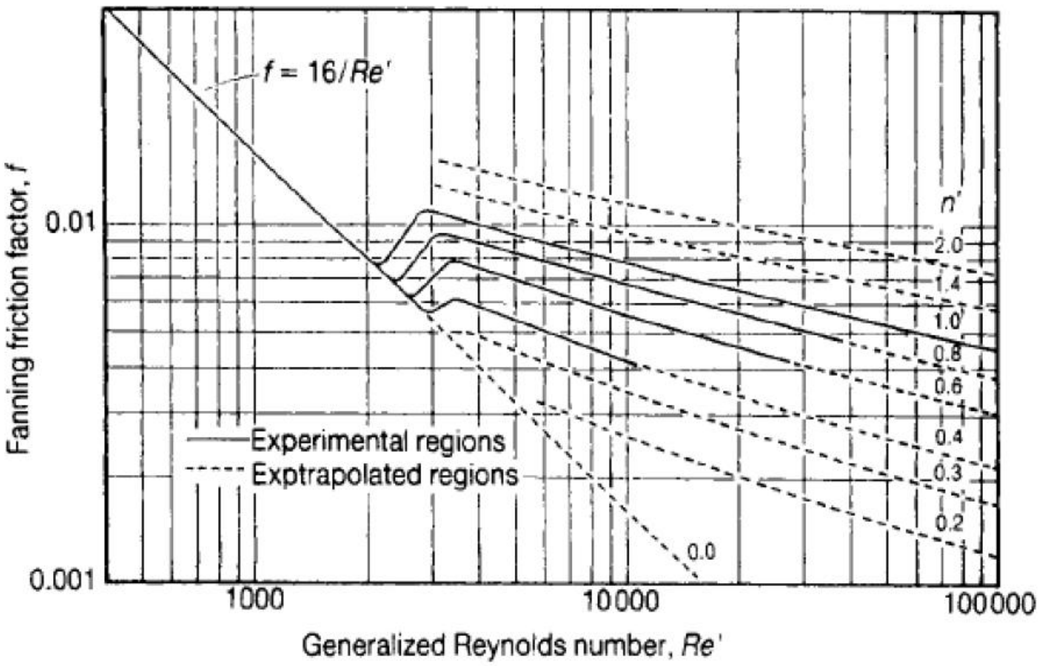

Figure of friction factor chart for purely viscous non-Newtonian fluids

Example -6.3-

A pseudoplastic fluid that follows the power-law, having a density of 961 kg/m^3 is flowing in steady state through a smooth circular tube having an inside diameter of 0.0508 m at an average velocity of 6.1 m/s. the flow properties of the fluid are n′ = 0.3, Kp = 2.744 Pa.sn. Calculate the frictional pressure drop across the tubing of 30.5 m long.

Solution:

$$Re = \frac{d^{\acute{n}} u^{2-\acute{n}} \rho}{m}$$

$= (1.523)^{0.3} (6.1)^{1.7} (961) / 2.744\ (8)^{-0.7}$

$= 1.328 \times 10^4$ ------------ the flow is turbulent

From Figure for Re = 1.328×10^4, n′ = 0.3 ⇒ $f = 0.0032$

$-\Delta P_{fs} = 4f\,(L/d)\,(\rho u^2/2) = 4(0.0032)\,(30.5 / 0.0508)[961(6.1)^2/2]$

⇒ $-\Delta P_{fs}$ = 134.4 kPa

Example -6.4-

The laminar flow velocity profile in a pipe for a power-law liquid in steady state flow is given by the equation

$$u_x = u\,\frac{3n+1}{n+1}\left[1-\left(\frac{2r}{d}\right)^{\frac{n+1}{n}}\right]$$

where n is the power-law index and u, is the mean velocity.

Use this equation to drive the following expression

$$\dot{\gamma}_w = -\left.\frac{du_x}{dr}\right|_{r=\frac{d}{2}} = \left(\frac{8u}{d}\right)\left(\frac{3n+1}{4n}\right)$$

Solution: $$u_x = u\,\frac{3n+1}{n+1}\left[1-\left(\frac{2r}{d}\right)^{\frac{n+1}{n}}\right]$$

$$\frac{du_x}{dr} = u\,\frac{3n+1}{n+1}\left[-\frac{n+1}{n}\,\frac{2}{d}\left(\frac{2r}{d}\right)^{\frac{n+1}{n}-1}\right] = \frac{2u}{d}\left(\frac{3n+1}{n+1}\right)\left(\frac{2r}{d}\right)^{\frac{1}{n}}$$

$$-\dot{\gamma}_w = -\left.\frac{du_x}{dr}\right|_{r=\frac{d}{2}} = \frac{2u}{d}\left(\frac{3n+1}{n}\right) \quad * \frac{4}{4}$$

$$-\dot{\gamma}_w = -\left.\frac{du_x}{dr}\right|_{r=\frac{d}{2}} = \frac{8u}{d}\left(\frac{3n+1}{4n}\right)$$

Home Work

P.6.1

The shear stress in power-law liquids in steady state laminar flow is given by the equation $\tau_{rx} = K\left(-\frac{du_x}{dr}\right)^m$,show that the velocity distribution is given by the following equation

$$u_x = u_{max}\left[1-\left(\frac{r}{R}\right)^{\frac{n+1}{n}}\right], where\ u_{max} = \frac{n}{n+1}\left(\frac{-\Delta P_{fs}}{2KL}\right)R^{\frac{n+1}{n}}$$

Hint: $\tau_{rx}(2\pi rL) = -\Delta P_{fs}(\pi r^2) \Rightarrow \tau_{rx} = \frac{-\Delta P_{fs}}{2L}\,r$

P.6.2

Calculate the frictional pressure gradient –ΔPfs/L for a time independent non-Newtonian fluid in steady state flow in a cylindrical tube if

the liquid density the	$\rho = 1000\ kg/m^3$
inside diameter of the tube	$d = 0.08\ m$
the mean velocity	$u = 1.0\ m / s$
the point pipe consistency coefficient	$K' = 2\ Pa.\ s^{0.5}$
and the flow behavior index	$n' = 0.5$.

P.6.3

Substitute the equation $\tau_{rx} = K\left(\frac{-du_x}{dr}\right)^n$ *into equation* $\frac{8u}{d} = \frac{32}{d^3}\int_0^{d/2} r^2\left(\frac{-du_x}{dr}\right)dr$

and integrate to show the shear rate at a pipe wall for power law fluid in steady state flow is $-\dot{\gamma}_w = -\left.\frac{du_x}{dr}\right|_{r=\frac{d}{2}} = \frac{8u}{d}\left(\frac{3n+1}{4n}\right)$

Hint: $\tau_{rx}(2\pi r L) = -\Delta P_{fs}(\pi r^2) \Rightarrow \tau_{rx} = \frac{-\Delta P_{fs}}{2L}\, r$